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PDE in strong form:

$$\operatorname{curl}\left(\frac{1}{\rho}\operatorname{curl} H\right) + \frac{\partial}{\partial t}\left(\mu(H)\cdot H\right) = 0 \qquad (1)$$

PDE in weak form:

$$\boxed{\int \frac{1}{\rho}\operatorname{curl} H \circ \operatorname{curl} v \; d\Omega + \int \frac{\partial}{\partial t}\left(\mu(H)\cdot H\right)\circ v \, d\Omega = 0} \qquad (2)$$

v = test function; In GetDP v is choosen $v = H$

Backward Euler:

$$\frac{\partial}{\partial t}\left(\mu(H)\cdot H\right) \longrightarrow \frac{\mu(H^n)\cdot H^n - \mu(H^{n-1})\cdot H^{n-1}}{\Delta t} \qquad (3)$$

with n: upper index for timestep

i: lower index for iteration number

$n \to$ actual time point
$n-1 \to$ known last time point

$i \to$ actual iteration
$i-1 \to$ known last iteration

		In GetDP
$\Rightarrow H_i^n$	$\to$ This we are looking for	Dof$\{H\}$
H_{i-1}^n	$\to$ Actual timepoint, but value from last iteration	$\{H\}$
H^{n-1}	$\to$ Value from last time step (Stored manually in Hpast in GetDP)	$\{Hpast\}$
Δt	$\to$ Time step size	\$DTime

Picard's Method

For the nonlinear part (μ in our case) use H from the last iteration:

$$\boxed{\int \frac{1}{\sigma} \operatorname{curl} H_i^n \circ \operatorname{curl} v \, d\Omega + \int \frac{\mu(H_{i-1}^n) \cdot H_i^n - \mu(H^{n-1}) \cdot H^{n-1}}{\Delta t} \circ v \, d\Omega = 0} \quad (4)$$

→ See Formulation Eddy_Formulation_H_Pic in Square.pro

Newton-Raphson Method

We want to set the following function f equal zero:

$$f(H^n) = \int \frac{1}{\sigma} \operatorname{curl} H^n \circ \operatorname{curl} v \, d\Omega + \int \frac{\mu(H^n) \cdot H^n - \mu(H^{n-1}) H^{n-1}}{\Delta t} \circ v \, d\Omega \overset{!}{=} 0 \quad (5)$$

Algorithm:

$$H_i^n = H_{i-1}^n - \frac{f(H_{i-1}^n)}{\frac{\partial}{\partial H_{i-1}^n} f(H_{i-1}^n)} \quad (6)$$

Defining $\delta H = H_i^n - H_{i-1}^n$ = change of H per iteration (7)

equation 6 can be rewritten as

$$\boxed{\frac{\partial}{\partial H_{i-1}^n} f(H_{i-1}^n) \cdot \delta H + f(H_{i-1}^n) = 0} \quad (8)$$

Note: In GetDP the word JacNL tells that Dof{H} means here the change of H in the iteration: !

$\delta H \longrightarrow$ JackNL[Dof{H} ...]

Equation (8) has to be implemented in the „Equation"-block.

$$f(H_{i-1}^n) = \int \frac{1}{\rho} \operatorname{curl} H_{i-1}^n \circ \operatorname{curl} v \, d\Omega + \int \frac{\mu(H_{i-1}^n) \cdot H_{i-1}^n - \mu(H^{n-1}) H^{n-1}}{\Delta t} \circ v \, d\Omega \qquad (9)$$

$$\frac{\partial}{\partial H_{i-1}^n} f(H_{i-1}^n) \cdot \delta H = \int \frac{1}{\rho} \operatorname{curl} \delta H \circ \operatorname{curl} v \, d\Omega + \int \frac{1}{\Delta t} \left(\mu(H_{i-1}^n) + \mu'(H_{i-1}^n) H_{i-1}^n \right) \delta H \circ v \, d\Omega \qquad (10)$$

Inserting (9) and (10) in (8) and doing a small simplification with (7) yield:

$$\frac{\partial}{\partial H_{i-1}^n} f(H_{i-1}^n) \cdot \delta H + f(H_{i-1}^n) \; \boxed{\begin{aligned} = & \int \frac{1}{\rho} \operatorname{curl} H_i^n \circ \operatorname{curl} v \, d\Omega \\ & + \int \frac{1}{\Delta t} \left(\mu(H_{i-1}^n) \cdot H_i^n - \mu(H^{n-1}) \cdot H^{n-1} \right) \circ v \, d\Omega \\ & + \int \frac{1}{\Delta t} \mu'(H_{i-1}^n) H_{i-1}^n \cdot \delta H \circ v \, d\Omega = 0 \end{aligned}} \qquad (11)$$

→ See Formulation Eddy_Formulation_H_NR

Newton-Raphson Method B-Version

Following the same procedure with

$$f(H^n) = \int \frac{1}{\rho} \operatorname{curl} H^n \circ \operatorname{curl} v \, d\Omega + \int \frac{B(H^n) - B(H^{n-1})}{\Delta t} \circ v \, d\Omega$$

gives:

$$f(H_{i-1}^n) = \int \frac{1}{\rho} \operatorname{curl} H_{i-1}^n \circ \operatorname{curl} v \, d\Omega + \int \frac{1}{\Delta t} \left(B(H_{i-1}^n) - B(H^{n-1}) \right) \circ v \, d\Omega$$

$$\frac{\partial}{\partial H_{i-1}^n} f(H_{i-1}^n) \cdot \delta H = \int \frac{1}{\rho} \operatorname{curl} \delta H \circ \operatorname{curl} v \, d\Omega + \int \frac{1}{\Delta t} B'(H_{i-1}^n) \cdot \delta H \circ v \, d\Omega$$

$$\frac{\partial}{\partial H_{i-1}^n} f(H_{i-1}^n) + f(H_{i-1}^n) \; \boxed{\begin{aligned} = & \int \frac{1}{\rho} \operatorname{curl} H_i^n \circ \operatorname{curl} v \, d\Omega \\ & + \int \frac{1}{\Delta t} \left(B(H_{i-1}^n) - B(H^{n-1}) \right) \circ v \, d\Omega \\ & + \int \frac{1}{\Delta t} B'(H_{i-1}^n) \cdot \delta H \circ v \, d\Omega = 0 \end{aligned}}$$

See Formulation
Eddy_Formulation_B_NR