Stationary Temperature Profile in a One - Dimensional Wire

We are looking for the Solution of the Differential Equation

$$\frac{d}{dx}\left(\lambda(T(x))\cdot\frac{d}{dx}T(x)\right) = -q_{dot} \qquad (1)$$

with the boundary conditions T(x0) = T0 ind T(x1)= T1

Without Current

Without current (q_{dot} = 0) equation (1) can be rewritten as a system of ordinary differential equations.
The unknown initial values T'(x0) can be obtained from the heat conduction integral:

$$\frac{\text{heat}}{\text{time}\cdot\text{area}} = j = \frac{1}{\text{length}}\cdot\int_{T0}^{T1}\lambda(T)\,dT \qquad \text{und} \qquad j(x0) = -\lambda(T0)\cdot\text{grad}(T(x0))$$

Since j = const along the wire we have j = j(x0)

$$T'_0 = \frac{d}{dx}T(x0) = \frac{1}{-\lambda(T0)\cdot\text{Länge}}\cdot\int_{T0}^{T1}\lambda(T)\,dT$$

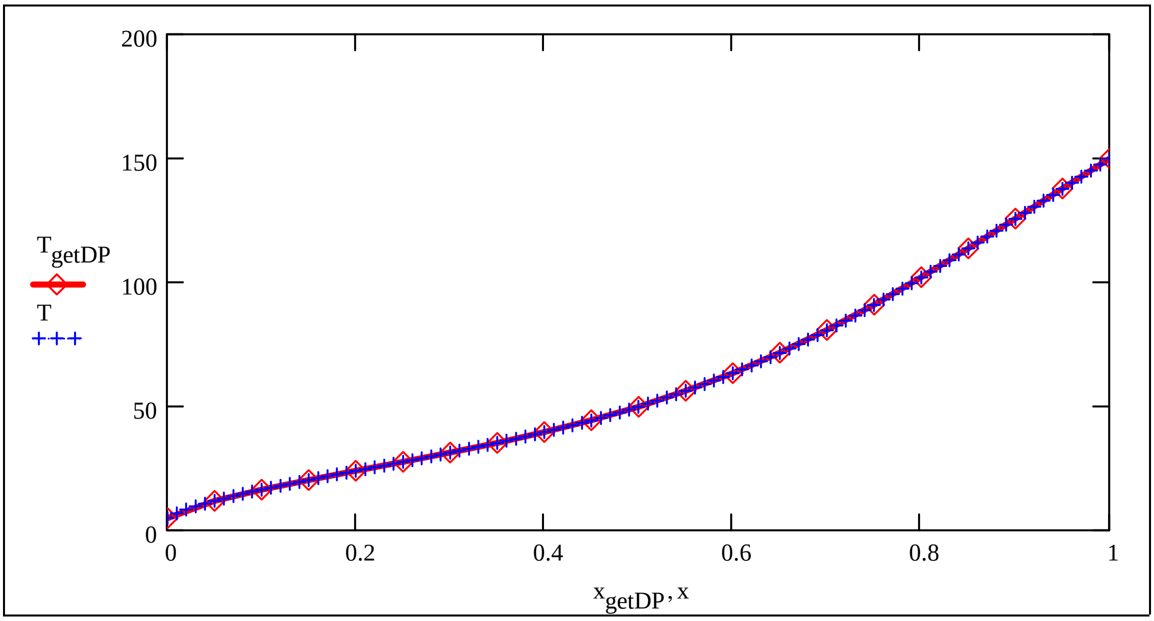
Thus we have two intital values for an ODE of 2nd order. This can be transformed into a system of ODE of 1st order which can be solved with simple numerical methods.

$$\tau_0 = T(x) \qquad \tau_1 = \frac{d}{dx}T(x) = \frac{d}{dx}\tau_0$$

combined with equation (1) yields:

$$\frac{d}{dx}\tau = \begin{bmatrix} \tau_1 \\ \frac{-\text{dlambda}(\tau_0)}{\text{lambda}(\tau_0)}\cdot(\tau_1)^2 \end{bmatrix}$$

Using the same materials data as in the getDP problem we get the following soution for a 1m wire and T(x0)=5K, T(x1)=150K :



with: $T_{x1} = 150.013$ and $\lambda_{x0} \cdot \frac{\Delta T}{\Delta x} = 1.046879 \times 10^{5}$

in agreement with the heat conduction integral: $j = -1.051387 \times 10^{5}$
(in W/m²)

With Current

The case with current in the lead allows no simple calculation of the initial values. Numerical tools need to be employed to obtain the values.
These suffer from severe limitations in convergence. To achieve convergence $\lambda(T)$ had to be fitted by an analytical expression which deviates more from the actual λ-data. Moreover the algorithm fails for currents higher than 0.327A. Nevertheless I compare this result to the getDP solution (obtained by using the same λ-function):