

Temperature Development in an infinite Quartz-Plate

See e.g.: Elsner, Grundlagen der Technischen Thermodynamik, Bsp. 11.5
(unfortunately only in German)

Non-stationary, one-dimensional temperature profile in an infinite plate subject to a sudden temperature change on it's surfaces (or in a 1-D wire/fibre neglecting surface effects like radiation coupling)

$d := 0.1 \cdot \text{m}$ Plate thickness

$\lambda := 1.45 \frac{\text{W}}{\text{m} \cdot \text{K}}$ $\rho := 2.2 \frac{\text{g}}{\text{cm}^3}$ $c := 700 \frac{\text{J}}{\text{kg} \cdot \text{K}}$ $a := \frac{\lambda}{\rho \cdot c}$ Materials properties

$t_a := 10 \text{ K}$ $t_0 := 100 \text{ K}$ $C(k) := (t_a - t_0) \cdot \frac{4 \cdot (-1)^{k+1}}{(2 \cdot k - 1) \cdot \pi}$

$$T(x, \tau) := t_0 + \sum_{k=1}^{100} C(k) \cdot e^{\frac{-(2 \cdot k - 1)^2 \cdot \pi^2 \cdot a \cdot \tau}{d^2}} \cdot \cos\left(\frac{2k - 1}{d} \cdot \pi \cdot x\right)$$

Solution for the left half of the plate:

