The heat transfer equations can be written as [1]:

$$\operatorname{div} \boldsymbol{q} + \rho c_p \, \partial_t T = p_q \quad \text{and} \quad \boldsymbol{q} = -\kappa \operatorname{grad} T \,, \tag{1a,b}$$

where $\boldsymbol{q}$ is the heat flux, ρc_p is the calorific capacity, T is the temperature, p_q is the heat source volume density and κ is the thermal conductivity. The heat equations may only be solved in the steel part, provided that proper convection and radiation conditions are imposed on its boundary.

The thermal formulation is directly written in terms of the temperature, with Newton-Robin boundary conditions describing the heat transfer by convection, i.e. $\boldsymbol{q}\cdot\boldsymbol{n}|_{\Gamma_{\text{conv}}} = \eta(T - T_0)$, and by radiation, i.e. $\boldsymbol{q}\cdot\boldsymbol{n}|_{\Gamma_{\text{rad}}} = \sigma\epsilon(T^4 - T_0^4)$ [2]. Since the emissivity depends not only on the temperature but also on the geometry of the considered structures, a more accurate approach involving an integral method [3] has also been implemented, but is not described here. A weak solution of (1) is obtained by solving:

$$\begin{aligned}(\kappa \operatorname{grad} T, \operatorname{grad} T')_\Omega + (\rho c_p \, \partial_t T, T')_\Omega - (p_q, T')_\Omega \\ + \langle \eta(T - T_0), T' \rangle_{\Gamma_{\text{conv}}} + \langle \epsilon\sigma_s(T^4 - T_0^4), T' \rangle_{\Gamma_{\text{rad}}} = 0 \,, \quad \forall T' \in F_T^0(\Omega) \,,\end{aligned} \tag{2}$$

where σ is Botltzmann's constant, ϵ is the emissivity, η is the convection coefficient, T_0 is the temperature of the materials surrounding the steel part, Ω is the thermal domain, and T and T' both belong to $F_T^0(\Omega)$. The operator $\langle \cdot, \cdot \rangle_\Gamma$ represents the surface integral on Γ of the product of its arguments. The temperature T and associated test functions T' are discretized with triangular nodal finite elements. The variation of the thermal conductivity, the calorific capacity and the emissivity are shown in Fig. 1. The convection coefficient η ranges from 2.5 to 5, while T_0 is set to $20\,^\circ$C.

References

[1] H. S. Carlaw and J. C. Jaeger. *Conduction of Heat in Solids.* Oxford University Press, 1959.

[2] R. Siegel and J. R. Howell. *Thermal Radiation Heat Transfer.* McGraw-Hill, 1972.

[3] E. M. Sparrow. *Radiation Heat Transfer.* Harpercollins College, 1978.

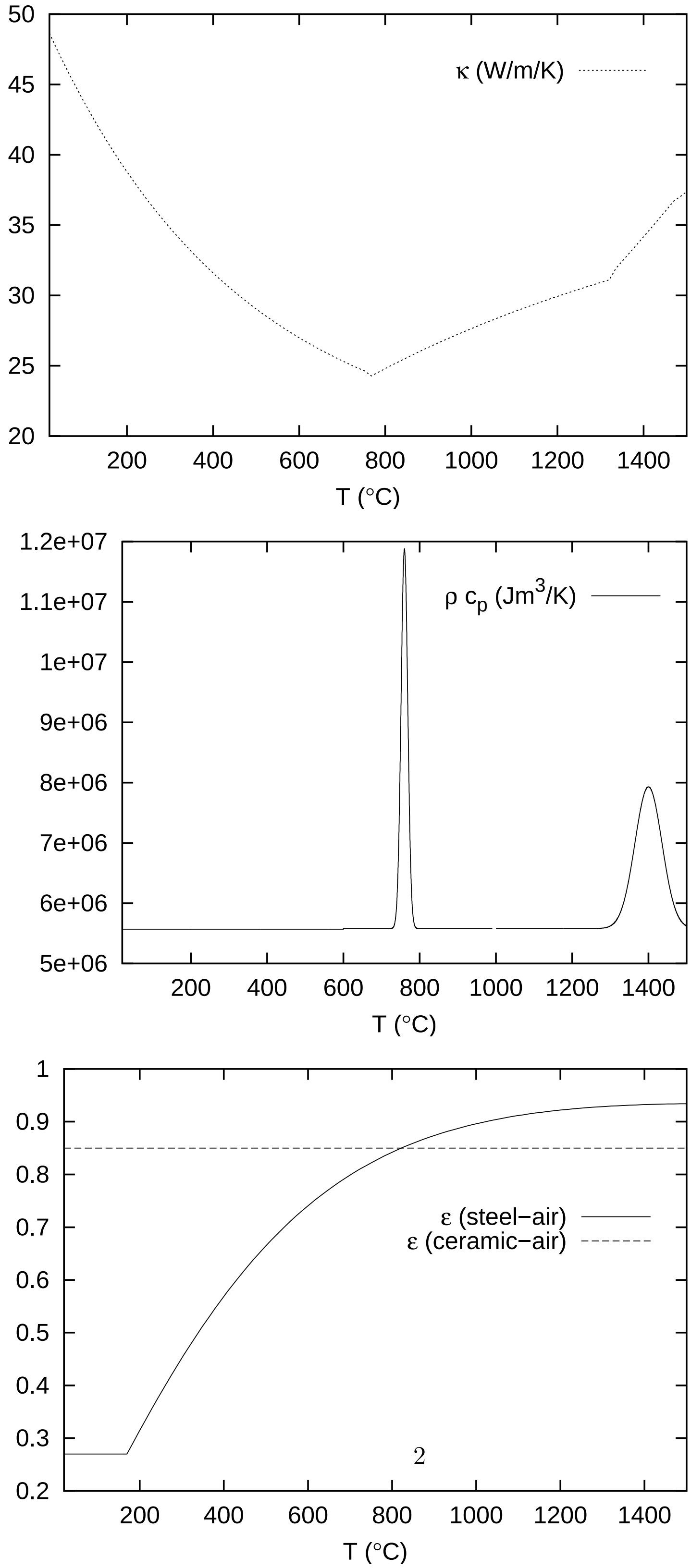

Figure 1: Thermal characteristics of the steel part.